MATH 464 (THEORY OF PROBABILITY)
HOMEWORK 8

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(1) Suppose X has density function $f_X(x) = c(1+x^2)$ for $-1 < x < 1$ and $f_X(x) = 0$ elsewhere.
   (a) Find the value of c.
   (b) Find the distribution function of X, i.e., $F_X(x)$.
   (c) Sketch the graphs of $f_X(x)$ and $F_X(x)$.
   (d) Compute $P(0 < X < 0.5)$.

(2) Consider $f(x) = cx^{-1/2}$ for $x \geq 1$, and $f(x) = 0$ otherwise. Show that there is no value of c that makes f a density function.

(3) Let X be a uniform random variable on the interval $(-1, 1)$. Find the distribution and density functions of $Y = |X|$. Is the distribution of Y in our catalogue of distributions? what is it?

(4) Let F_1 and F_2 be distribution functions of some random variables, show that for every $0 \leq \alpha \leq 1$, the function

$$F = \alpha F_1 + (1-\alpha)F_2$$

is a distribution function of some random variable.

(5) Suppose X is an exponential random variable with parameter $\lambda = 1$. Find the distribution and density functions of $Y = \ln(X)$.
Note: This is called the *double exponential* distribution.

(6) For any $\omega > 0$, let

$$\Gamma(\omega) := \int_0^\infty x^{\omega-1} e^{-x} dx.$$

Show that if ω is a positive integer then $\Gamma(\omega) = (\omega - 1)!$

(7) Find the distribution of the so-called "extreme value" density function

$$f(x) = \exp(-x - e^{-x}) \text{ for } x \in \mathbb{R}.$$